Assignment = 5

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Sub:- Introduction to Engineering Mathematics with Application

Q=1] State Stokes Theorem for Convergence.

A= Stokes Theorem:- If c is a Simple Closed Curve Which Enclosed a Surface S and $\vec{F}(x, y, z)$ is a vector function which is Continuous at every point of S Then

$$\oint \vec{F} \cdot d\vec{r} = \int_S (\nabla \times \vec{F}) \cdot \hat{n} \, ds$$

Q=2] Find the Curl of the vector Field $\vec{P} = x^2\hat{i} + y\hat{j} + 2z\hat{k}$ at the point $(1, 2, 2)$

S= Curl $\vec{P} = \nabla \times \vec{P}$ $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & y & 2z \end{vmatrix}$

$$\hat{i} \begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & 2z \end{vmatrix} - \hat{j} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ x^2 & 2z \end{vmatrix} + \hat{k} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ x^2 & y \end{vmatrix}$$

$$\hat{i}(0-0) - \hat{j}|0-0| + \hat{k}|0-0|$$

$= 0$ Ans

Q=3] Find the Directional Derivative of $f(x, y, z) = x + yz^3$ in the Direction of the vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ at the point $(0, -1, 1)$

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8= Given $f(x,y,z) = x + yz^3$; $P = (0,-1,1)$

$$\vec{u} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$|\vec{u}| = \sqrt{2^2+3^2+4^2}$$
$$= \sqrt{4+9+16} = \sqrt{29}$$

$$|\vec{u}| = \sqrt{29}$$

Formula - $\boxed{\dfrac{\nabla f|_P \cdot \vec{u}}{|\vec{u}|}}$

$$\nabla f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z} = \hat{i}$$

$$\nabla f = (1+0)\hat{i} + (0+z^3)\hat{j} + (0+3yz^2)\hat{k}$$

$$\nabla f = \hat{i} + \hat{j}z^3 + \hat{k}\,3yz^2$$

then point, $x=0$, $y=-1$, $z=1$

$$\nabla f|_{(0,-1,1)} = \hat{i} + \hat{j} + \hat{k}\,3(-1)(1)^2$$

$$\nabla f|_{(0,-1,1)} = \hat{i} + \hat{j} - 3\hat{k}$$

$$= \frac{(\hat{i}+\hat{j}-3\hat{k}).(2\hat{i}+3\hat{j}+4\hat{k})}{\sqrt{29}}$$

$$\cancel{3\hat{i}+3} \frac{2+3-12}{\sqrt{29}}$$

$$= \frac{2-9}{\sqrt{29}}$$

$$= \frac{-7}{\sqrt{29}} \quad \underline{Ans}$$

Q=4] If V is the volume enclosed by closed surface S and $\vec{f} = 2x\hat{i} + y\hat{j} + z\hat{k}$ find

$\iint \vec{f}.\hat{n}\,ds$. and S is the surface of the sphere

$x^2+y^2+z^2=1$

So= $$\iiint_V \nabla.\vec{f}\,dv = \iiint\left(\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}\right)dv$$

$$= \iiint_V (2+1+1)\,dv =$$

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$$= 4 \iiint_V dv$$

$$= 4 \times \frac{4}{3} \pi (1)^3$$

$$= \frac{16}{3} \pi (1)^3 \quad \underline{\text{Ans}}$$

Q=5] If f is a homogenous function of degree n, then Justify.

$$x^2 \frac{\partial^2 f}{\partial x^2} + y^2 \frac{\partial^2 f}{\partial y^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} = n(n-1) f.$$

S= We know that

Euler's 1st order equation :-

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = nf$$

Partial differentiation w to x.

$$\frac{\partial f}{\partial x} + x \frac{\partial^2 f}{\partial x^2} + y \frac{\partial^2 f}{\partial x \partial y} = n \frac{\partial f}{\partial x}$$

$$= x \frac{\partial^2 f}{\partial x^2} + y \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial x} (n-1) \text{ ——— } (1)$$

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Partial differtiation w.r.to y

$$= x\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial f}{\partial y} + y\frac{\partial^2 f}{\partial y^2} = n\frac{\partial f}{\partial y}$$

$$= x\frac{\partial^2 f}{\partial x \partial y} + y\frac{\partial^2 f}{\partial y^2} = \frac{\partial f}{\partial y}(n-1) \text{ — Eq-} 2$$

Multiplication of x in Eq-1 and y in Eq-2

$$= x^2\frac{\partial^2 f}{\partial x^2} + yx\frac{\partial^2 f}{\partial x y} + xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2} =$$

$$(n-1)\left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\right) \text{ — } 3$$

Put $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = nf$ from Eq-A in Eq-3

$$= x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2} = n(n-1)f$$

Hence Proved

Ans

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Completed